Abstract

The calculation of the thermodynamic properties for humid air as a real gas mixture using the virial equation of state is presented. The investigations include the Hyland and Wynder model (1983) and consider the Nelson and Sauer model (2001). All of the latest NIST standards for the properties of dry air and virial coefficients and formulations of the International Associations for the Properties of Water and Steam (IAPWS) for the properties of ice, for sublimation pressure, and for Henry's constant have been incorporated. The range of validity of the proposed model is in pressure from 0.01 kPa to 10 MPa, in temperature from 130 to 623.15 K, and in humidity ratio from 0 to 10 kg(water)/kg(dry air). This model devotes only slightly from the Hyland-Wynder and Nelson-Sauer models at ambient pressure, the differences increase with increasing pressure and temperature. The developed algorithms have been implemented in a library for practical use. In addition to thermodynamic properties, data, backward functions on various sets of independent variables and transport properties can be calculated. The property library can be used for calculating air-conditioning processes, compressed air storage processes and other thermal processes having the working fluid humid air.

Methodology

The properties of humid air are calculated from the modified Hyland-Wynder model. The modifications consist in:

- Value for the universal molar gas constant \( \mathcal{R} \) from the CODATA standard by Mele and Taylor (2005).
- Value for the molar mass of dry air \( M_{\text{a}} \) from Galaty and et al. (2000) and that of ice from IAPWS-06 (IAPWS 2006).
- Molar volume for humid air can be calculated iteratively from the expression

\[
\frac{V}{\text{mol}} = \frac{\mathcal{R} T}{p} \left(1 + \frac{B}{2} \frac{p}{\mathcal{R} T} + \frac{C}{6} \frac{p^2}{\mathcal{R} T^2} + \cdots \right)
\]

where \( p \) is the total pressure, \( \mathcal{R} \) is the universal molar gas constant, \( M_{\text{a}} \) is the molar mass of dry air, and \( \mathcal{R} T \) is the temperature in Kelvin.

**Proposed Algorithms**

**Virtual Equation of State**

The mixture humid air is calculated using the following mixing virtual equation of state. The equation contains virtual coefficients up to the third virial coefficient. The virtual equation of state reads

\[
\frac{p}{\mathcal{R} T} = \sum_{n=0}^{N} f(n) \left( \frac{p}{\mathcal{R} T} \right)^n
\]

where \( p, T \) are the total pressure and temperature, \( \mathcal{R} \) is the universal molar gas constant, and \( f(n) \) are the virial coefficients from the property library. For dry air \( f(n) = 0 \) for all \( n > 0 \).

**Molar Volume**

The mole fraction of water vapor obtained from the humidity ratio \( \phi \) via

\[
\phi = \frac{p_{w}}{p} = \frac{x_{w}}{x_{a}} = \frac{1}{1 + \frac{B_{w}}{2} x_{w} + \cdots}
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where \( x_{a} \) is the mole fraction of water vapor obtained from the humidity ratio \( \phi \) via

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**Saturation Composition**

The mole fraction of water in saturated humid air is

\[
\phi_{w} = \phi_{w,s}
\]

Functions of the Property Library

The algorithms for the properties of humid air described in this paper have been implemented into the property library LibHuAirProp (Kretzschmar et al. 2009), which can be received from the authors by request (h.kretzschmar@itw.rwth-aachen.de).

Table 1: Properties of the property library LibHuAirProp

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References


Zittau/Goerlitz University of Applied Sciences

**Thermodynamic Properties of Real Moist Air**

Kretzschmar, H.-J., Herrmann, S., and Gatley, D. P.

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**Range of Validity**

The proposed algorithms, which are included into the property library LibHuAirProp, can be used in the following ranges of temperature and pressures:

\[
T = 273.15 \text{ K} \quad \text{to} \quad 623.15 \text{ K}
\]

and

\[
p = 0.01 \text{ kPa} \quad \text{to} \quad 10 \text{ MPa}
\]

with the limitation that the partial pressure of steam is restricted to 16 MPa and some small variations for transport-property calculations. The humidity ratio of humid air can range from \( \phi = 0 \) (dry air) to \( \phi = 10 \text{ kg(water)/kg(dry air)} \).

Figure 1: Pressure-temperature diagram with arrangement of the ranges for steam, water, and ice.

Figure 2: Values of the virtual-pressure enhancement factor calculated from the shown equation for several total pressures plotted over temperature T.